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MOMENTUM ANALYSIS OF TORNADO WIND ENERGY CONCENTRATOR SYSTEMS

E. DICK
Department of machinery
State University of Gent
Sint Pietersnieuwstraat 41
9000 GENT, BELGIUM

ABSTRACT

The momentum theory is applied to the tornado wind energy concentrator system. It is shown that the power coefficient of the system can be written as a product of three factors : a mass concentration coefficient, an energy augmentation coefficient and an extraction coefficient. By this splitting, the relative importance of the three components in the power output of the system can be analysed. On small scale systems, including a real turbine, the three coefficients are determined and inherent drawbacks of the tornado system are detected. Extrapolation of the characteristic coefficients for large scale systems is possible leading to a performance prediction for real applications.

KEYWORDS - Tornado wind energy system   Momentum theory   Mass concentration coefficient   Energy augmentation coefficient   Extraction coefficient

INTRODUCTION

The Tornado Wind Energy System (TWES) was introduced by Yen [1,2]. Figure 1 shows the concept. The main part of the system is a tower which collects the wind through adjustable vanes that are opened at the windward side and closed at the leeward side. A vortex is created in the tower with a low pressure core that sucks air through a turbine in the bottom of the tower.

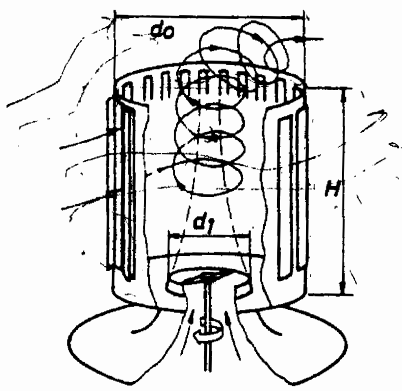

Fig. 1

Global performance predictions for the tornado system, based on moment considerations were formulated by Loth [3] and by Dick [4]. A general momentum theory for wind energy concentrator systems, separating the characteristics of the turbine and the concentrator parts was formulated by Dick [5]. In this paper, this general momentum theory is applied to the tornado wind energy system.

MOMENTUM THEORY FOR FREE TURBINES

In order to be able to apply momentum theory to a tornado system, it is necessary to recall the classic momentum theory of Betz for free turbines [6].

Figure 2 shows the flow through a wind turbine, schematically represented by an actuator disk. The undisturbed flow far upstream of the wind turbine has velocity v_0. The wind velocity at the turbine is v_1. Far downstream of the wind turbine, the velocity is v_2. The air is assumed to be incompressible with density ρ. Due to the energy extraction by the turbine, the wind velocity v_2 is smaller than v_0. Hence, the streamtube through the turbine diverges. Some of the retardation of the flow takes place in front of the turbine, some behind the turbine. A uniform static pressure drop $p^+ - p^-$ over the turbine expresses the energy extraction. In the wake, the static pressure recovers to the free stream value $p_2 = p_0$. The pressure profile is sketched in figure 2.

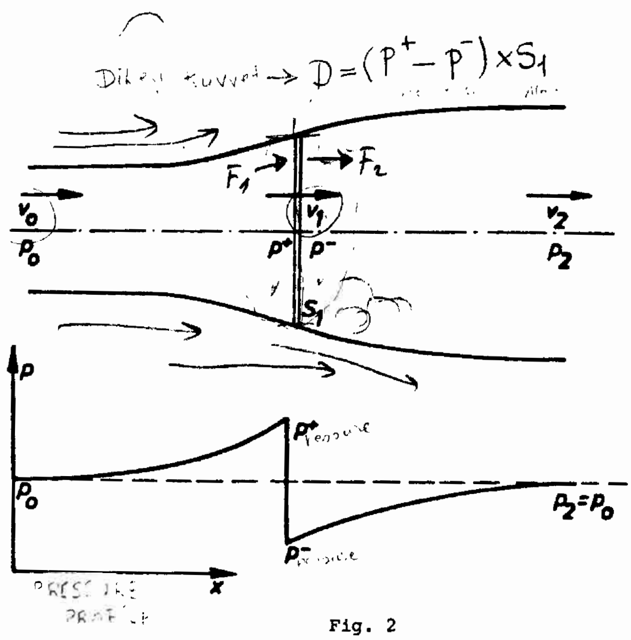

Fig. 2

From the law of conservation of momentum, it follows that the axial force $D = (p^+ - p^-)S_1$ is equal to the decrease of the momentum flux over the turbine.

Assuming inviscid flow, there cannot be an exchange of enthalpy without mass transfer. Hence, the region influenced by the wind turbine is limited to the streamtube through the turbine. The decrease in momentum is thus :

$$\rho v_1 S_1 (v_0 - v_2) = D = (p^+ - p^-) S_1 \tag{1}$$

Since there is no enthalpy exchange upstream and downstream of the turbine, the Bernoulli equation holds for these parts of the streamtube :

$$p^+ + \frac{1}{2}\rho v_1^2 = p_0 + \frac{1}{2}\rho v_0^2$$

$$p^- + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

Since $p_2 = p_0$, this leads to :

$$p^+ - p^- = \frac{1}{2}\rho (v_0^2 - v_2^2) \tag{2}$$

(203-173)

The total enthalpy drop across the turbine is

$$\Delta H = \frac{p^+ - p^-}{\rho} = \frac{1}{2}\,(v_0^2 - v_2^2)$$

Hence, the power extracted by the turbine is

$$P = \rho v_1 S_1 \Delta H = \frac{1}{2}\,\rho v_1 S_1 (v_0^2 - v_2^2)$$

The power can be expressed by the power coefficient

POWER COEFFICIENT:

$$\boxed{C_P = \frac{P}{\frac{1}{2}\,\rho S_1 v_0^3} = \left(\frac{v_1}{v_0}\right)\left[1 - \left(\frac{v_2}{v_0}\right)^2\right]} \qquad (3)$$

Combination of (1) and (2) gives

$$v_1 = \frac{v_0 + v_2}{2} \qquad \text{or} \qquad \frac{v_2}{v_0} = 2\,\frac{v_1}{v_0} - 1 \qquad (4)$$

Hence, half the retardation of the flow takes place upstream and half the retardation takes place downstream of the turbine.

The power coefficient expresses the ratio of the power extracted by the turbine to the energy flux in a streamtube in undisturbed flow with a cross section equal to the turbine through flow surface S_1. Hence, the expression

$$\frac{v_1}{v_0}\left[1 - \left(\frac{v_2}{v_0}\right)^2\right] \qquad (5)$$

can be seen as an extraction coefficient, indicating the part of the energy extracted from the flow which passes through the turbine in unloaded conditions.

: EXTRACTION FACTOR

With the relationship (4), the extraction factor (5) has a maximum for $v_1/v_0 = 2/3$, equal to

BETZ Limit: $C_{P,max} = 0.593$ (6)

Bütün mesele ... yeni design ...

BASIC DEFINITIONS

Clearly, due to the upper limit of the extraction factor (6) and the rather limited energy flux which can pass in unloaded conditions through a free turbine, the power of a free turbine is rather limited. The aim of concentrator systems is to alleviate the power limit of a wind energy system. Principally, this can be done in three ways, which usually are combined : augmenting the mass flow passing through the system, augmenting the specific energy content of the wind and augmenting the energy extraction factor.

In unloaded conditions, for a free turbine, the velocity at the turbine (v_{10}) is equal to v_0. By augmentation of the mass flow through the system, it is to be understood that in unloaded conditions the velocity at the turbine is made to be larger than the free stream velocity. A mass concentration coefficient then can be defined by [5] :

CONCENTRATION COEFFICIENT: $C_c = v_{10}/v_0$ (7)

From (2), it is seen that for a free turbine, the maximum enthalpy drop is the kinetic energy content of the free stream (reached for $v_2 = 0$) :

$$\Delta H_m = v_0^2/2$$

It is said that an energy augmentation effect is achieved if this maximum enthalpy drop is made larger than the free stream kinetic energy. An energy augmentation coefficient is defined by [5] :

ENERGY AUGMENTATION COEFFICIENT: $C_a = 2\Delta H_m/v_0^2$ (8)

The absolute maximum power of the system is then :

$$\boxed{P_{max} = \rho v_{10} S_1 \Delta H_m}$$

and an extraction coefficient can be defined by :

EXTRACTION COEFFICIENT: $C_e = P/P_{max}$ (9)

Clearly, the power coefficient of the system is given by :

$$C_P = \frac{P}{\rho v_0\,\frac{1}{2}\,v_0^2 S_1} = \frac{\rho v_{10}}{\rho v_0}\,\frac{2\Delta H_m}{v_0^2}\,\frac{P}{P_{max}} = C_c C_a C_e \qquad (10)$$

POWER COEFFICIENT

It is clear a priori that mass concentration is a matter of exerting forces perpendicular to the flow, i.e. forces which do not transfer energy to the flow. Obviously, in order to reach an energy augmentation effect, forces which execute work have to be exerted on the flow.

The result of the Betz theorem (6) cannot be interpreted as an absolute result, since it is obtained by a number of approximations. The theory is an example of a streamtube theory. This means that the energy extraction by the turbine is calculated based on momentum balances, on streamtubes, neglecting enthalpy exchange between flows in different streamtubes. This is clearly an inviscid approximation.

In loaded conditions, there is always a spontaneous energy transfer to the wake of the system. It arises from turbulent mixing with surrounding air which is not affected by the system itself. This turbulent mixing leads to the wake decay. For free turbines, the phenomenon of turbulent mixing was discussed by Hütter [7]. According to his measurements, the average axial velocity in the streamtube through the turbine can be sketched as in figure 3. The axial velocity at the turbine is higher than predicted by inviscid streamtube theory. Downstream of the turbine, the axial velocity reaches a value that is slightly lower than predicted.

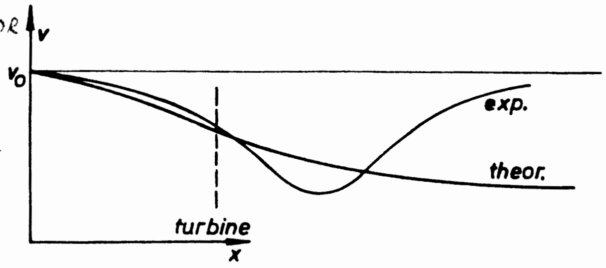

Fig. 3

Except for very low values of v_2, the functional relation between the minimal axial velocity v_2 and the axial velocity at the turbine v_1 can be approximated by

$$\frac{v_2}{v_0} = 2\,\frac{v_1}{v_0} - 1 - 0.69\,\left(\frac{v_1}{v_0} - 1\right)^2 \qquad (11)$$

Neglecting the energy transfer to the wake due to turbulent mixing, before the point of minimum axial velocity, the power extracted by the turbine is still given by equation (3). With equation (11), the power coefficient reaches a maximum for

$$v_1/v_0 = 0.676 \qquad v_2/v_0 = 0.280$$

POWER COEFFICIENT:

leading to

$$C_{P,max} = 0.623$$

$0{,}676 \times [1-(0{,}28)^2] = [1-0{,}0784] = [\ldots 1216 \times 0{,}676] \quad C_{P,max} = 0{,}623$

This shows that, by the mixing of the wake and the surrounding flow, an increase in extractable energy may be expected in the order of 5 %.

For low values of v_2/v_0, it is not possible to determine a clear functional relationship between v_2/v_0 and v_1/v_0 due to the breakdown of the regular flow pattern with the occurence of a turbulent wake state.

In practice, the energy transfer to the wake due to turbulent mixing can be taken into account by not measuring v_2 but by calculating it from the axial force coefficient of the turbine :

$$\boxed{C_D = \frac{D}{\frac{1}{2}\,\rho v_0^2 S_1} = \frac{p^+ - p^-}{\frac{1}{2}\,\rho v_0^2}}$$

AXIAL FORCE COEFFICIENT

The aim of Concentrator systems:
is to alleviate the power limit of a wind energy system:
1) Augmenting the mass flow passing through the system:
2) Augmenting the specific energy content of the wind
3) Augmenting the energy extraction factor:

From (2), it follows

$$\boxed{C_D = 1 - (\frac{v_2}{v_0})^2} \qquad (12)$$

SYSTEM FUNCTION

Since the difference between v_2 calculated by (12) and the actual v_2 downstream of the turbine is limited to a few percent, values of C_D larger than 1 only occur in turbulent wake state, i.e. in load conditions which are far away from optimum. Hence by using (12), it is not necessary to incorporate the energy augmentation effect due to turbulent mixing into the energy augmentation coefficient. To do so would be very difficult since this effect is not determined by the system itself. Therefore, in the sequel, v_2 always will be defined by (12). The functional relationship between v_2 and v_1, obtained in this way, will be called the system function.

MOMENTUM THEORY FOR THE TORNADO WIND ENERGY SYSTEM

A lot of studies, both analytically and experimentally, were done on the vortex structure in the tornado tower. A thorough analytical study can be found in [8], an experimental study on towers with closed bottom walls in [9], experimental studies on towers in which the turbine is replaced by a screen in [10] and [11] and an experimental study with a real turbine in [12]. All these studies agree that the vortex in the tower can be seen as composed of a core region ($0 < r < r_c$) with approximately solid body rotation and an outer region ($r_c < r < r_0$) with approximately potential vortex structure. The tangential velocity profile can be well represented by a Rankine-law :

RANKINE LAW

$$\boxed{w = (K/r)(1 - \exp(-Re\ r^2/2\ r_0^2))} \qquad (13)$$

in which v_0 is the free stream velocity and Re is a Reynolds number. This structure is asymptotically limited by an ideal solid body rotation in the core :

$$w = w_m\ (r/r_c) \qquad (14)$$

and an ideal potential vortex in the outer region :

$$w = K/r \qquad \text{with} \qquad w_m = K/r_c \qquad (15)$$

Experimentally, the tangential velocity at $r = r_0$ is found to be slightly lower than the free stream velocity v_0. Therefore, it is believed that a theoretical upper limit of K is $v_0 r_0$. In the core region, pressure is approximately constant. In the outer region of the vortex, centrifugal force and pressure gradient are in equilibrium [9] :

$$\frac{1}{\rho}\frac{dp}{dr} = \frac{w^2}{r} \qquad (16)$$

For $r = r_0$, pressure is approximately equal to the free stream pressure p_0. Measured tangential velocity - and pressure profiles are shown in figure 4b [10].

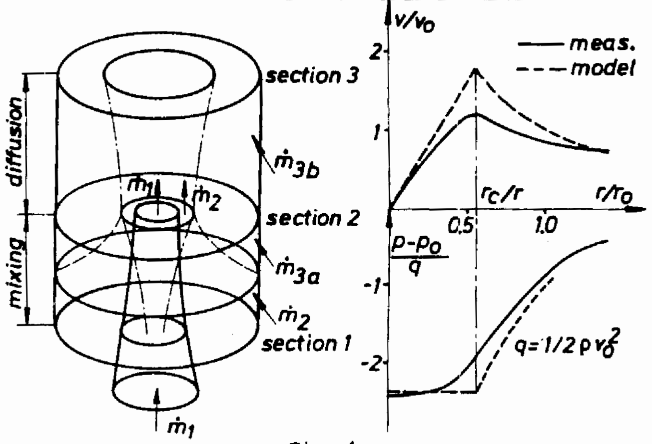

Fig. 4

According to the measurements described above, the tangential velocity profile can be modelled by (14) (15) with $K = v_0 r_0$. With the potential vortex structure (15), the pressure in the outer region follows from (16) :

$$\frac{1}{\rho}(p_0 - p) = \frac{v_0^2}{2}\left[(\frac{r_0}{r})^2 - 1\right] \qquad (17)$$

The result (17) implies that the total enthalpy in the potential region of the vortex is equal to the free stream value. This shows that there is no energy exchange between the flow in the potential region and the turbine and that the measured decrease in total enthalpy with decreasing radius in the potential region is due to losses. As a consequence, only the core region plays a role in the energy exchange between flow and turbine.

The complete flow pattern is sketched in figure 4a. In the lower part of the tower the Rankine-like shape of the tangential velocity profile is not yet established due to the mixing of the turbine flow and some of the flow that enters through the vanes. The lowest horizontal cross section in which the mixing is completed, i.e. in which the Rankine-law can be detected, is called the mixed-out section (section 2). The turbine flow is called the primary flow. The flow through the vanes which is collected in the core region of section 2 is called the secondary flow. The flow which comes in the potential region of section 2 and which enters in higher parts of the tower, is called the tertiary flow.

The ratio of secondary to primary mass flow can be determined theoretically by expressing conservation of moment of momentum between section 1 and section 2. In case the turbine is replaced by a screen, the moment of momentum just behind the turbine can be neglected and conservation of moment of momentum, based on the model flow, gives :

$$\int_0^{r_{c2}} \rho v_2 w 2\pi r dr + \int_{r_{c2}}^{R} \rho v K 2\pi r dr = K(\dot{m}_2+\dot{m}_3), \quad K = v_0 r_0$$

The axial velocity in the core region in section (2) is denoted by v_2 while v is the axial velocity in the potential region. By assuming v_2 to be constant :

$$\frac{1}{2}K(\dot{m}_1+\dot{m}_2) + K\dot{m}_3 = K(\dot{m}_2+\dot{m}_3) \qquad \text{or} \qquad \dot{m}_2/\dot{m}_1 = 1 \qquad (18)$$

The result (18) only holds for the model flow. In a real system, the secondary mass flow is lower than the value predicted by (18).

It is to be remarked that the mass flow ratio can be influenced by the moment of momentum in the primary flow. By denoting $K_T\dot{m}_1$ the flux of moment of momentum in the turbine flow, the balance (18) becomes :

$$\frac{1}{2}K(\dot{m}_1+\dot{m}_2) + K\dot{m}_3 = K(\dot{m}_2+\dot{m}_3) + K_T\dot{m}_1 \quad \text{or} \quad \frac{\dot{m}_2}{\dot{m}_1} = 1 - \frac{2K_T}{K} \qquad (19)$$

Clearly, by giving a postrotation in the turbine flow, opposite to the sense of rotation of the vortex in the tower, the secondary mass flow can be increased.

Due to the mixing between secondary and primary flow, an energy transfer between both flows is possible. By assuming that this energy transfer occurs without losses, an upper limit of the power of the turbine can be found from an energy balance on the core flow between section 2 and a free stream position for both the primary and secondary flow [4].

From measurements [10] it is clear that the pressure which is maintained in the mixed-out section, in loaded conditions of the turbine, is below ambient pressure by an amount which is much higher than the dynamic pressure in this section. This means that the core flow has to recover to ambient pressure while energy is transfered from the tertiary flow by mixing. It is clear that this combined diffusion-mixing process actually determines the pressure that can be maintained in the mixed-out section. By assuming that the energy transfer from the tertiary flow occurs without losses, an upper limit for the power of the turbine can be found from an energy balance between section (3) and free stream positions for all flows involved [3].

The distinction between secondary and tertiary mass flows is very fundamental in the understanding of the energy transfer processes in the tornado system. Detailed measurements by Windrich and Fricke [11] show that the mixed-out section is an equilibrium section without radial flow and without axial pressure gradient. A consequence of this observation is that the flow field in the mixed-out section is twodimensional, such that the tangential velocity profile and the pressure profile in the mixed-out section are independent of turbine load and tower exit conditions. It is clear that this only can be valid, if the velocity and pressure field in the mixed-out section can freely develop. This implies a sufficient height of the tower. This was clearly demonstrated in [10] in which towers with free suction are used in contrast to the towers in [11] in which by means of a blower the pressure in front of the screen is kept at atmospheric pressure, independent of turbine load. In [10] it was shown that the diffusion process between mixed-out section and tower top requires a minimum axial length in order to avoid return flow. For aspect ratios H/d_0 higher than 2.5, a regular diffusion, involving only positive axial velocities is observed, leading to a well developed mixed-out section with a pressure profile that is twodimensionally determined and is independent of tower height. As a consequence, for tower aspect ratios higher than 2.5, the power that can be extracted by the turbine becomes independent of tower height. For aspect ratios lower than 2.5 return flow in the axial diffusion is observed leading to a decrease of the subpression in the core of the mixed-out section and a decrease in extractable power. As a consequence, based on frontal area, the power coefficient reaches a maximum for the aspect ratio 2.5, i.e. at the onset of return flow, independent of other system parameters.

The existence of a mixed-out section, observed experimentally, can be used to simplify the theoretical analysis of the tornado system. Since, at least for sufficiently high aspect ratios, the pressure profile which is maintained in the mixed-out section is twodimensionally determined, i.e. only dependent on the form of the cross section of the tower and its diameter, independent of load, the turbine flow can be considered to exhaust in a constant pressure reservoir similar to an infinite atmosphere. This makes it possible to extend classical momentum theory for free turbines to tornado concentrators as was principally shown in [5]. In this section, the momentum analysis of the tornado concentrator is further developed based on the concepts that were introduced in [5] for general concentrator systems. In contrast to [3] and [4], in the sequel, momentum theory is used in the classical sense of Betz. This means that the energy balance is taken on the streamtube passing through the turbine between mixed-out section 2 and free stream conditions, neglecting energy transfer due to mixing. A correction for mixing can be introduced, as shown in the previous section, by calculating v_2 from the axial force coefficient of the turbine by (12).

When energy transfer in the streamtube through the turbine is neglected upstream of the turbine and between turbine and mixed-out section, the Bernoulli equation holds for these parts of the streamtube :

$$p^+ + \frac{1}{2}\rho v_1^2 = p_0 + \frac{1}{2}\rho v_0^2 \qquad\qquad p^- + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

Hence : $$p^+ - p^- = p_0 + \frac{1}{2}\rho v_0^2 - \frac{1}{2}\rho v_2^2 - p_2 \qquad (20)$$

In contrast to the free turbine $p_2 \neq p_0$, and as a consequence, in unloaded conditions, v_{20} is different from v_0 :

$$\frac{1}{2}\rho v_{20}^2 = p_0 + \frac{1}{2}\rho v_0^2 - p_2 \qquad (21)$$

Hence, according to the definition (8), the energy augmentation coefficient is :

$$C_a = v_{20}^2/v_0^2 \qquad (22)$$

Using (20) and (21), the power extracted by the turbine is given by :

$$\boxed{P = \rho v_1 S_1 \frac{1}{2}(v_{20}^2 - v_2^2)}$$

The power coefficient is :

$$C_P = \frac{P}{\frac{1}{2}\rho v_0^3 S_1} = \frac{v_1}{v_0}\left[\left(\frac{v_{20}}{v_0}\right)^2 - \left(\frac{v_2}{v_0}\right)^2\right] \qquad (23)$$

Using the energy augmentation (22) and the mass concentration coefficient (7), (23) becomes :

$$C_P = \left(\frac{v_{10}}{v_0}\right)\left(\frac{v_{20}}{v_0}\right)^2\left(\frac{v_1}{v_{10}}\right)\left[1 - \left(\frac{v_2}{v_{20}}\right)^2\right] \qquad (24)$$

Hence, the extraction coefficient has the form :

$$\boxed{C_e = \frac{v_1}{v_{10}}\left[1 - \left(\frac{v_2}{v_{20}}\right)^2\right]} \qquad (25)$$

It is clear that (25) is a generalisation of (5), since for the free turbine $v_{10} = v_0$ and $v_{20} = v_0$.

From (20), it is seen that v_2/v_{20} can be calculated from the measured force coefficient of the turbine :

$$C_D = \frac{p^+ - p^-}{\frac{1}{2}\rho v_0^2} = C_a\left[1 - \left(\frac{v_2}{v_{20}}\right)^2\right] \qquad (26)$$

(26) is to be considered as a generalisation of (12).

EXPERIMENTAL VERIFICATION

It is clear that the quality of the vortex structure is strongly dependent on the geometry of the vanes. In order to circumvent the problem of designing optimal vanes, for analysis purposes a logarithmic spiral shaped tower was introduced by Yen [1,2] in which the ideal vortex structure (13,14) is easily reached. Results for models of this type with closed bottom walls were reported by Windrich et al. [9]. The same logarithmic spiral shape was used by Haers and Dick [10] in studies on towers with a bottom opening in which the turbine is simulated by a screen. A similar study was done by Windrich and Fricke [11]. In [12] also results are reported on cilindrical towers with vanes.

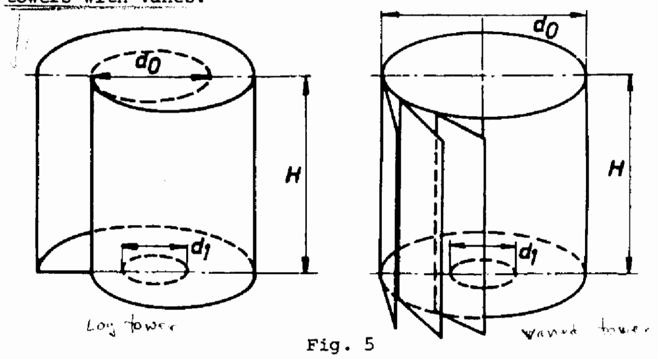

Fig. 5

Figure 5 shows the logarithmic spiral tower used in [9-12]. The form of the spiral is :

$$\boxed{r = r_0\, e^{.1\,\theta}} \qquad 0 \leq \theta \leq 2\pi$$

The logarithmic spiral tower is to be seen as a tornado tower with diameter d_0 and one, large, vane collecting the wind in an optimal way. Therefore, the frontal area of the system is calculated as $H\ d_0$. Figure 5 also shows the model of the cilindrical tower used in [12]. Vanes are opened in one quarter of the tower.

Figure 6 shows the power coefficient, based on frontal area for the logarithmic tower and the cilindrical tower for several values of tower aspect ratio H/d_0 and turbine diameter to tower diameter ratio d_1/d_0, in function of screen porosity a. This figure shows that the performance of the omni-directional tower is consistently lower than the performance of the uni-directional